Grey Verhulst Model Based on BP Neural Network Optimization for Oil Production Forecasting

Deqiang Zhou

School of Information and Mathematics, Yangtze University

Jingzhou, Hubei, China

zdqmfk@yahoo.com.cn

***Abstract*-The grey Verhulst model has been widely applied in oil production forecasting. However, there are many defects of parameters estimation in traditional grey Verhulst model. To overcome the defects, an optimized grey Verhulst model based on BP neural network is proposed. Firstly, the Verhulst model is mapped to a BP neural network, the corresponding relationships between grey Verhulst model parameters and BP network weights is established. Then, the BP neural network is trained by means of BP algorithm, when the BP network convergences, the optimized weights can be extracted, and the optimized grey Verhulst model based on BP algorithm can be realized. By applying this optimized grey Verhulst model to the national oil production forecasting, the experiment results show that the method is feasible and effective, especially, when the oil production increases according to the curve with S-type, not only higher forecasting accuracy can be obtained, but also the superiority and the features of grey system model can be reserved.**

Keywords-Oil ProductionForecasting; Grey Verhulst Model; BP Neural Network; Data Fitting; Optimized Modeling

I. INTRODUCTION

With the fast economic development, the energy demand is increasing year after year, especially the oil demand. The supply of oil is a hot problem. The condition of oil reserves in china is not good. Most oil field has been at the stage of mining metaphase or mining anaphase. The increase of oil production is very difficult. Facing the rising oil demand increasingly, the forecast of oil production scientifically and exactly in the future has realism significance to the plan of oil demand and oil stratagem repertory [1, 2]. There are many methods for oil production forecasting, and the grey system model is one of them. The GM (1, 1) model is widely discussed and studied in the grey system. The purpose of GM (1, 1) model is to work on system forecasting with poor, incomplete or uncertain messages. In the prediction of oil production, we usually use the grey GM (1, 1) model. However, it is imperfect when the oil production increases in the curve with S-type, or the increment of oil production is in the saturation stage. In this case, the oil production forecasting error of grey system model will become larger and the result is unaccepted in the real world. The grey Verhulst model is a special kind of model within the grey system. The grey Verhulst model [2-9] is used to forecast the raw sequence growth in S-type or growth being saturated. Therefore, in recent years this model has been widely applied in oil production forecasting, especially, when the oil production increases according to the curve with S-type [1, 2, 3]. However, there are many defects of parameters estimation in traditional grey Verhulst model [2-8].First, the traditional model need to face the reasonable selection of background value. On the other hand, in order to obtain forecasting model, in these improved models, parameters which obtained from the grey difference equation is substituted into the grey differential equation, thus we also can not avoid the jumping errors from the difference equation to differential equation in traditional grey modeling [2, 10-12]. This parameter estimation method which obtains through the indirect method may not guarantee that the value and the actual value of the gray prediction error are minimum. Generally, from the evaluation criteria of the model fitting, when solving grey Verhulst model model, the model should directly take minimizing the error of the predicted value and the actual value as the criterion.

To consider the forecasting of the raw sequence growth in S-type or growth being saturated, from the data fitting's viewpoint, this study uses BP neural network model to overcome the defects of grey Verhulst model. The grey system's primitive discrete data is fitted by a continuous model [13] which has the same form with grey Verhulst model's time response type, the model parameters are trained and optimised by means of BP neural network, and then the optimization grey Verhulst model is realized.

The remainder of this paper is organized as follows. In Section 2, the grey Verhulst model is introduced. In Section 3, the Verhulst model is mapped to a BP neural network, the relationships between rey Verhulst model parameters and BP network weights is established, where BP network algorithm is used to optimize grey Verhulst model parameters. The experimental results and discussions are presented in Section 4. Finally, conclusion is drawn in Section 5.

II. GREY VERHULST MODEL

Grey Verhulst model constructing process is described below:

Denote the nonnegative original sequence by

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)). \quad (1)$$

The accumulated generating operation (AGO) [1] formation of $X^{(0)}$ is defined as:

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)), \quad (2)$$

where $x^{(1)}(1) = x^{(0)}(1), x^{(1)}(k) = \sum_{j=1}^{k} x^{(0)}(j), \quad k = 2, 3, \cdots, n.$

The grey Verhulst model can be established by constructing a first order differential equation for $X^{(1)}$

$$\frac{\mathrm{d}x^{(1)}}{\mathrm{d}t}+ax^{(1)}=b(x^{(1)})^2 \tag{3}$$

The solution of Eq.(3)can be obtained by using the least square method, that is:

$$\hat{x}^{(1)}(k+1)=\frac{\hat{a}x^{(0)}(1)}{\hat{b}\hat{x}^{(0)}(1)+(\hat{a}-bx^{(0)}(1))\mathrm{e}^{ak}} \tag{4}$$

where

$$[\hat{a},\hat{b}]=(B^TB)^{-1}B^TY \text{ and } B=\begin{bmatrix}-z_2^{(1)} & (z_2^{(1)})^2\\ -z_3^{(1)} & (z_3^{(1)})^2\\ \vdots & \vdots\\ -z_n^{(1)} & (z_n^{(1)})^2\end{bmatrix} \quad Y=\begin{bmatrix}x_2^{(0)}\\ x_3^{(0)}\\ \vdots\\ x_n^{(0)}\end{bmatrix};$$

$$z_k^{(1)}=0.5(x^{(1)}(k)+x^{(1)}(k-1))\cdot$$

Applying the inverse accumulated generating operation (IAGO) [1], the grey Verhulst model fitted and predicted values are obtained by

$$\hat{x}^{(0)}(k)=x^{(1)}(k+1)-x^{(1)}(k) \quad (k=2,3,\cdots)$$

$$\hat{x}^{(0)}(1)=x^{(0)}(1)\,. \tag{5}$$

Assume that the original sequence $X^{(0)}$ itself increases in the curve with S-type or the increment of the original sequence is in the saturation stage, the original sequence can be taken as $X^{(1)}$, the IAGO of the original sequence can be taken as $X^{(0)}$.

III. GREY VERHULST MODEL BASED ON BP NEURAL NETWORK

Assume that $X^{(0)}$ increases in the curve with S-type or the increment of the original sequence is in the saturation stage, to consider the following non-linear function,

$$y(t)=\frac{a\hat{x}^{(0)}(1)}{b\hat{x}^{(0)}(1)+(a-bx^{(0)}(1))e^{a(t-1)}}\ . \tag{6}$$

where $a,b,\hat{x}^{(0)}(1)$ are undetermined parameters.

Compared with grey Verhulst model, the function value of formula (6)at t （$t=2,3,\cdots,n$） can be taken as the predicted value in the corresponding time t of the grey Verhulst model. In order to obtain a better solution, this initial value does not take the first data of the original sequence data, but as a parameter to be determined by data fitting. Hence, the original discrete data sequence $X^{(0)}$ can be fitted by formula (6). The key question is how to estimate the parameters by original discrete data $X^{(0)}$.

Because the error function is nonlinear, we now resolve the above problem by means of BP network which can approximate nonlinear function with any precision [14].

First, the formula (6) is transformed as follows:

$$y(t)=\frac{1}{\frac{b}{a}+(\frac{1}{\hat{x}^{(0)}(1)}-\frac{b}{a})e^{a(t-1)}} \tag{7}$$

$$\frac{1}{y(t)}=\frac{b}{a}+(\frac{1}{\hat{x}^{(0)}(1)}-\frac{b}{a})e^{a(t-1)} \tag{8}$$

Since $e^{-a(t-1)}>0$, therefore,

$$\begin{aligned}\frac{1}{y(t)}&=\left((\frac{1}{\hat{x}^{(0)}(1)}-\frac{b}{a})\frac{e^{-a(1-t)}}{(1+e^{-a(1-t)})}+\frac{b}{a}\frac{1}{1+e^{-a(1-t)}}\right)(1+e^{-a(1-t)})\\ &=\left((\frac{1}{\hat{x}^{(0)}(1)}-\frac{b}{a})\left(1-\frac{1}{(1+e^{-a(1-t)})}\right)+\frac{b}{a}\frac{1}{1+e^{-a(1-t)}}\right)(1+e^{-a(1-t)})\\ &=\left((\frac{1}{\hat{x}^{(0)}(1)}-\frac{b}{a})-\frac{1}{x^{(0)}(1)}\frac{1}{(1+e^{-a(1-t)})}+\frac{2b}{a}\frac{1}{1+e^{-a(1-t)}}\right)(1+e^{-a(1-t)}),\end{aligned} \tag{9}$$

The above equation is mapped to the BP neural network, the network structure is described as Fig.1:

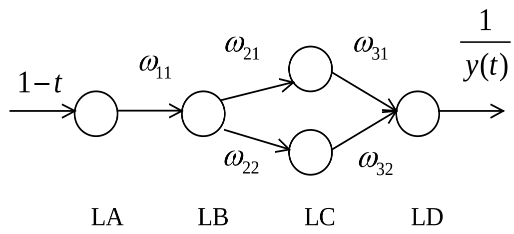

Figure1 Grey BP neural network structure

The corresponding relationships between the network weight and the grey Verhulst model parameters are established as follows:

$$\omega_{11}=a,\quad \omega_{21}=-\frac{1}{\hat{x}^{(0)}(1)},$$

$$\omega_{22}=\frac{2b}{a},\quad \omega_{31}=\omega_{32}=1+e^{-a(1-t)} \tag{10}$$

The threshold value of LD level is

$$\theta=(1+e^{-a(1-t)})(\frac{b}{a}-\frac{1}{\hat{x}^{(0)}(1)})\cdot$$

And the LB level neuron's transfer function is taken as Sigmoid type functions $f(x)=\dfrac{1}{1+e^{-x}}$, there is a high-gain area which ensure that the network eventually reach a steady [13].Other neuron's transfer function is taken as an e linear function $f(x)=x$.

Through the above method, the grey Verhulst model is mapped to a BP neural network; figure 1 shows this neural network structure for grey Verhulst model is simple. The corresponding relation between the grey Verhulst model parameters and BP network weights is established, then the grey Verhulst model parameters estimation problem is transformed into the weights of neural network optimization problem. The BP neural network is trained by means of data sets $(1-t,\dfrac{1}{x^{(0)}(t)}),t=1,2,\cdots,n$, when the BP network convergence, the optimized model parameters can be obtained [13], and the optimization modeling for the grey Verhulst model based on BP algorithm can be realized. The model can avoid the jumping errors from the difference equation to differential equation, and overcome the defects of the grey Verhulst model.

IV. EXPERIMENT RESULTS AND DISCUSSIONS

A. Experiment Design

In this section, to validate the performance of the grey Verhulst model based on BP neural network. The optimized grey Verhulst is applied for national oil production forecasting, the primary data is shown in Table Ⅰ [1].

TABLE I. NATIONAL OIL PRODUCTION FROM 2002 TO 2009 [UNIT: MILLION TONS]

Year	2002	2003	2004	2005	2006	2007	2008	2009
Serial number	0	1	2	3	4	5	6	7
Production	16.70	16.95	17.58	18.13	18.47	18.63	19.00	18.94

The actual value data presents the saturation characteristic; therefore, it is suitable for grey Verhulst modeling. According to the proposed method in this paper, the BP neural network with three hidden layer is established. The network structure is described as Fig.1. The first level neuron's transfer function is taken Log-Sigmoid type function; other neuron's transfer function is taken as a linear function. The maximum training epoch is $M = 300$, the allowance permissible error is $E = 0.00000005$. The LM (Levenberg -Marquardt) is taken as the training algorithm; the learning rate is dynamically determined by LM algorithm. To use the Matlab6.5 programming, when the network achieves the accuracy requirement, we can obtain the optimized grey Verhulst BP neural network model.

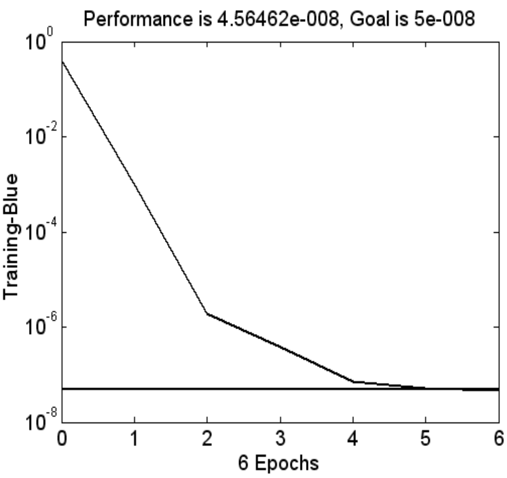

Figure2 Training errors

Fig.2 shows the training times of this method only use 6 steps to reach the minimum error. However, the traditional BP network not only need a large amount of data to train, but the theoretical guidance of network settings is also lack, moreover the training time is much greater than this article method [14], Many experiments show that the network convergence epochs of the method is generally about 10. Experiment shows the method combines the characteristics of the small sample and poor information of the Verhulst model, and displays the characteristics of strong nonlinear approximation and fault-tolerant capability of the neural network.

B. Empirical Results and Error Analysis

Compared with the traditional grey Verhulst model and other method in the paper [1], the fitted value of different models and the actual value are shown in Fig. 3 for comparison.

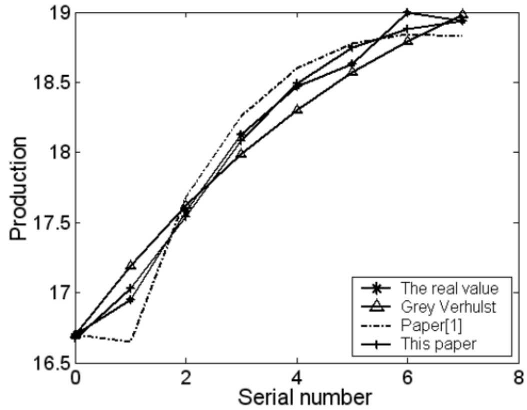

Figure3 Production of crude oil from 2002 to 2009 fitted by different models

Mean absolute percentage error (MAPE) approach [15] has been recommended to validation. Mean absolute percentage error is defined as

$$\mathrm{MAPE} = \frac{1}{l}\sum_{i=1}^{l}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \tag{11}$$

where y_i is the actual value, and $\hat{y}_i$ is the predicted value, $\left|\frac{y_i - \hat{y}_i}{y_i}\right|$ is absolute percentage error of y_i. The Experiment results are shown in Tab. Ⅱ.

TABLE II. THE FORECASTING RELATIVE ERRORS BASED ON DIFFERENT MODELS

Model	**Grey Verhulst model**	**Paper[1]**	**This paper**
MAPE %	0.76	0.75	0.32

According to results listed in Tab. Ⅱ, compared with the traditional model, the precision in this article enhances57.89%. Compared with other method in the paper [1], the precision in this article enhances 57.33%. Analyzing the errors of competitive models by means of comparing the MAPE, we conclude that the method in this paper is a better fitted prediction model. Furthermore, according the criteria purposed by Lewis [15]. The model is better than the other forecasting models because of lower MAPE.

V. CONCLUSION

1. From the data fitting's viewpoint, the grey Verhulst model can be optimized by means of BP algorithm.

2. The optimized grey Verhulst model in this paper can overcome the defects of the grey Verhulst.

3. The experiment results show that the optimized grey Verhulst for oil production forecasting is applicable, especially, when the oil production increases according to the curve with S-type, not only higher forecasting accuracy can be obtained, but also the superiority and the features of grey system model can be reserved.

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